Class XII Session 2025-26
Subject - Mathematics
Sample Question Paper - 3

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. If A' $= \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$, then (A + 2B)' is **[1]**

   a) $\begin{bmatrix} 1 & -4 \\ 6 & 5 \end{bmatrix}$ b) $\begin{bmatrix} -4 & 5 \\ 1 & 6 \end{bmatrix}$

   c) $\begin{bmatrix} -4 & 6 \\ 1 & 5 \end{bmatrix}$ d) $\begin{bmatrix} -4 & 1 \\ 5 & 6 \end{bmatrix}$

2. Matrices A and B are inverses of each other only when **[1]**

   a) AB = BA = O b) AB = O, BA = I

   c) AB = BA d) AB = BA= I

3. The value of the determinant $\begin{vmatrix} 2 & 7 & 1 \\ 1 & 1 & 1 \\ 10 & 8 & 1 \end{vmatrix}$ is: **[1]**

5. If the points A(-1, 3, 2), B(-4, 2, -2) and C(5, 5, λ) are collinear then the value of λ is **[1]**

   a) 47 b) 49

   c) -51 d) -79

4. If y = 2^x then $\frac{dy}{dx} = ?$ **[1]**

   a) $x(2^{x-1})$ b) 2^x (log 2)

   c) $\frac{2^x}{(\log 2)}$ d) 3^x (log 3)

   a) 10 b) 7

   c) 5 d) 8

6. The degree of the differential equation $x^2 \frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^3$ is **[1]**

a) 2
b) 1
c) 3
d) 6

7. The corner points of the feasible region for a Linear Programming problem are P(0, 5), Q(1, 5), R(4, 2) and S(12, 0). The minimum value of the objective function Z = 2x + 5y is at the point. **[1]**

a) R
b) Q
c) S
d) P

8. The principal value of the expression $\cos^{-1}$ [cos (– 680°)] is **[1]**

a) $\frac{\pi}{9}$
b) $\frac{2\pi}{9}$
c) $\frac{-2\pi}{9}$
d) $\frac{34\pi}{9}$

9. $\int_{-\pi}^{\pi} \sin^5 x dx = ?$ **[1]**

a) 2π
b) $\frac{3\pi}{4}$
c) 0
d) $\frac{5\pi}{16}$

10. If $\begin{bmatrix} 2x+y & 4x \\ 5x-7 & 4x \end{bmatrix} = \begin{bmatrix} 7 & 7y-13 \\ y & x+6 \end{bmatrix}$, then the value of x, y is **[1]**

a) x = 3, y = 1
b) x = 2, y = 3
c) x = 2, y = 4
d) x = 3, y = 3

11. If the constraints in a linear programming problem are changed **[1]**

a) the problem is to be re-evaluated
b) solution is not defined
c) the objective function has to be modified
d) he change in constraints is ignored

12. Let $\vec{a}, \vec{b}, \vec{c}$ be three unit vectors such that $|\vec{a} + \vec{b} + \vec{c}| = 1$ and $\vec{a}$ is perpendicular to $\vec{b}$. If $\vec{c}$ makes angle α and β with $\vec{a}$ and $\vec{b}$ respectively, then cos α + cos β = **[1]**

a) $-\frac{3}{2}$
b) 1
c) -1
d) $\frac{3}{2}$

13. If P = $\begin{vmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{vmatrix}$ is the adjoint of a 3 $\times$ 3 matrix A and |A| = 4, then α is equal to **[1]**

a) 11
b) 4
c) 5
d) 13

14. The letters of the word ORIENTAL are arranged in all possible ways. The chance that the consonants and vowels occur alternately is **[1]**

a) $\frac{3}{35}$
b) $\frac{1}{35}$
c) $\frac{2}{35}$
d) $\frac{1}{70}$

15. If x = a cos θ + b sin θ, y = a sin θ - b cos θ, then which one of the following is true? **[1]**

a) $y^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$
b) $y^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} - y = 0$

c) $y^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=0$

d) $y^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0$

16. Which one of the following is the unit vector perpendicular to both $\vec{a}=-\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}-\hat{j}+\hat{k}$? **[1]**

a) $\frac{\hat{j}+\hat{k}}{\sqrt{2}}$

b) $\pm\frac{\hat{i}+\hat{j}}{\sqrt{2}}$

c) $\frac{\hat{i}-\hat{j}}{\sqrt{2}}$

d) $\hat{k}$

17. Let f(x) = x - |x| then f(x) is **[1]**

a) differentiable $\forall$ x $\in$ R

b) continuous $\forall$ x $\in$ R and not differentiable at x = 0

c) discontinuous at x = 0

d) neither continuous nor differentiable at x = 0

18. If a line makes angles $\alpha,\beta,\gamma,\delta$ with four diagonals of a cube then $\cos^2\alpha+\cos^2\beta+\cos^2\gamma+\cos^2\theta$ is equal to **[1]**

a) $\frac{2}{3}$

b) $\frac{4}{3}$

c) $\frac{8}{3}$

d) $\frac{1}{3}$

19. **Assertion (A):** If x is real, then the minimum value of x^2 - 8x + 17 is 1. **[1]**

Reason (R): If f''(x) > 0 at a critical point, then the value of the function at the critical point will be the minimum value of the function.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** The modulus function f : R $\rightarrow$ R given by f(x) = |x| is neither one-one nor onto. **[1]**

Reason (R): The signum function f : R $\rightarrow$ R given by f(x) = $\begin{cases}1, & x>0\\ 0, & x=0\\ -1, & x<0\end{cases}$ is bijective.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. $\sin^{-1}\left(\frac{-1}{2}\right)$ **[2]**

OR

Find the value of $\cos^{-1}\frac{1}{2}+2\sin^{-1}\frac{1}{2}$.

22. Show that the function given by f (x) = sin x is decreasing in $\left(\frac{\pi}{2},\pi\right)$ **[2]**

23. A particle moves along the curve y = $(\frac{2}{3})x^3$ + 1. Find the points on the curve at which the y-coordinate is changing twice as fast as the x-coordinate. **[2]**

OR

The total revenue in Rs received from the sale of x units of the product is given by $R(x)=13x^2+26x+15$. find Marginal Revenue when 17 unit are produce.

24. Evaluate: $\int e^x\cdot\left\{\frac{2+\sin 2x}{1+\cos 2x}\right\}dx$. **[2]**

25. If $A=\begin{bmatrix}1 & 1 & -2\\ 2 & 1 & -3\\ 5 & 4 & -9\end{bmatrix}$ find |A|. **[2]**

Section C

26. Evaluate: $\int \frac{3\sin x+2\cos x}{3\cos x+2\sin x}dx$ **[3]**

27. The contents of three bags I, II and III are as follows: **[3]**

Bag 1 : 1 white, 2 black and 3 red balls,

Bag II : 2 white, 1 black and 1 red ball;

Bag III : 4 white, 5 black and 3 red balls.

A bag is chosen at random and two balls are drawn. What is the probability that the balls are white and red?

28. Evaluate $\int \frac{2x}{(x^2+1)(x^2+3)}dx.$ **[3]**

OR

Evaluate: $\int_0^{\pi/2} \frac{1}{1+\cot x}dx$

29. $(x^2 + y^2)$ dy = xydx. If y(1) = 1 and y (x_0) = e, then find the value of x_0. **[3]**

OR

Show that the differential equation $(y^2 - x^2)$dy = 3xydx is homogeneous and solve it.

30. If $\vec{a}+\vec{b}+\vec{c}=\overrightarrow{0}$, prove that $(\vec{a}\times\vec{b})=(\vec{b}\times\vec{c})=(\vec{c}\times\vec{a})$. **[3]**

OR

Using vectors, find the area of the ΔABC, whose vertices are A (1, 2, 3), B(2, -1, 4) and C(4, 5, -1).

31. If $x\sqrt{1+y}+y\sqrt{1+x}=0$ and x $\neq$ y, prove that $\frac{dy}{dx}=-\frac{1}{(x+1)^2}$. **[3]**

Section D

32. Find the area enclosed by the parabola $4y = 3x^2$ and the line 2y = 3x + 12. **[5]**

33. Let R be a relation on $N\times N$, defined by (a, b) R (c, d) $\Leftrightarrow$ a + d = b + c for all (a, b), (c, d) $\in N\times N$. Show that R is an equivalence relation. **[5]**

OR

Show that the function $f: R\rightarrow\{x\in R: -1<x<1\}$ defined by $f(x)=\frac{x}{1+|x|}$, x $\in$ R is one-one and onto function.

34. Express the matrix A = $\begin{bmatrix}1 & 3 & 5\\ -6 & 8 & 3\\ -4 & 6 & 5\end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix. **[5]**

35. AB is the diameter of a circle and C is any point on the circle. Show that the area of $\triangle$ABC is maximum, when it is an isosceles triangle. **[5]**

OR

Show that the height of a closed right circular cylinder of given surface and maximum volume is equal to diameter of base.

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

To hire a marketing manager, it's important to find a way to properly assess candidates who can bring radical changes and has leadership experience.

Ajay, Ramesh and Ravi attend the interview for the post of a marketing manager. Ajay, Ramesh and Ravi chances of being selected as the manager of a firm are in the ratio 4 : 1 : 2 respectively. The respective probabilities for them to introduce a radical change in marketing strategy are 0.3, 0.8, and 0.5. If the change does take place.

i. Find the probability that it is due to the appointment of Ajay (A). (1)

ii. Find the probability that it is due to the appointment of Ramesh (B). (1)

iii. Find the probability that it is due to the appointment of Ravi (C). (2)

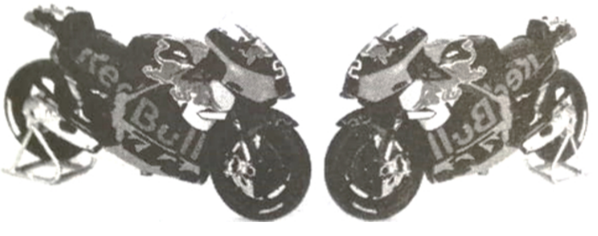
OR

Find the probability that it is due to the appointment of Ramesh or Ravi. (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

A motor cycle race was organized in a town, where the maximum speed limit was set by the organizers. No participant are allowed to cross the specified speed limit, but Two motorcycles A and B are running at the speed more than allowed speed on the road along the lines

$\vec{r} = \lambda(\hat{i} + 2\hat{j} - \hat{k})$ and $\vec{r} = 3\hat{i} + 3\hat{j} + \mu(2\hat{i} + \hat{j} + \hat{k})$, respectively.



i. Find the Cartesian equation of the line along which motorcycle A is running. (1)

ii. Find the shortest distance between the lines. (1)

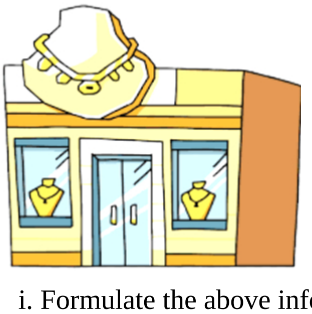
iii. If the direction cosines of a lines are $\frac{k}{3}, \frac{k}{3}, \frac{k}{3}$, then find the value of k. (2)

OR

The point where the line joining the points (0, 5, 4) and (1, 3, 6) meets XY - plane. (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

Dinesh is having a jewelry shop at Green Park, normally he does not sit on the shop as he remains busy in political meetings. The manager Lisa takes care of jewelry shop where she sells earrings and necklaces. She gains profit of ₹30 on pair of earrings & ₹40 on neckless. It takes 30 minutes to make a pair of earrings and 1 hour to make a necklace, and there are 10 hours a week to make jewelry. In addition, there are only enough materials to make 15 total of jewelry items per week.



i. Formulate the above information mathematically. (1)

ii. Graphically represent the given data. (1)

iii. To obtain maximum profit how many pair of earing and neckleses should be sold? (2)

OR

What would be the profit if 5 pairs of earrings and 5 necklaces are made? (2)

Solution

Section A

1.

(b) $\begin{bmatrix} -4 & 5 \\ 1 & 6 \end{bmatrix}$

Explanation:

$A' = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$

$\Rightarrow$ A = (A')' $= \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix}$

$\therefore$ A + 2B $= \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix} + 2\begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$

$= \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} -2 & 0 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 1 \\ 5 & 6 \end{bmatrix}$

$\Rightarrow$ (A + 2B)' $= \begin{bmatrix} -4 & 5 \\ 1 & 6 \end{bmatrix}$

2.

(d) AB = BA= I

Explanation:

A = B^{-1}

B = A^{-1}

We know that

AA^{-1} = I

(Given B = A^{-1})

AB = I ...(i)

We know that

BB^{-1} = I

(Given A = B^{-1})

BA = I ...(ii)

From (i) and (ii)

AB = BA = I

3. **(a)** 47

Explanation:

47

$\begin{vmatrix} 2 & 7 & 1 \\ 1 & 1 & 1 \\ 10 & 8 & 1 \end{vmatrix}$

= 2(1-8)-7(1-10)+1(8-10)

= 2(-7) - 7(-9) +1(-2)

= -14 +63-2

= -16 +63

= 47

4.

(b) 2^x (log 2)

Explanation:

Given that $y = 2^x$

Taking log both sides, we get

$\log_e y = x \log_e 2$ (Since $\log_a b^c = c \log_a b$)

Differentiating with respect to x, we get

$\frac{1}{y}\frac{dy}{dx} = \log_e 2$ or $\frac{dy}{dx} = \log_e 2 \times y$

Hence $\frac{dy}{dx} = 2^x \log_e 2$

5. **(a)** 10

Explanation:

Determinant of these point should be zero

$$\begin{vmatrix} -1 & 3 & 2 \\ -4 & 2 & -2 \\ 5 & 5 & \lambda \end{vmatrix} = 0$$

$-1(2\lambda + 10) - 3(-4\lambda + 10) + 2(-20 - 10) = 0$

$10\lambda = 10 + 30 + 60 = 100$

$\lambda = 10$

6.

(b) 1

Explanation:

$$x^2 \frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^3$$

$$x^3 \frac{d^2y}{dx^2} = x^3\left(\frac{dy}{dx}\right)^3 - y^3 - 3x^2y\left(\frac{dy}{dx}\right)^2 + 3y^2x\frac{dy}{dx}$$

$$x^3 \frac{d^2y}{dx^2} - x^3\left(\frac{dy}{dx}\right)^3 + 3x^2y\left(\frac{dy}{dx}\right)^2 + 3xy^2\frac{dy}{dx} + y^3 = 0$$

Order $= 2$

degree $= 1$

7. **(a)** R

Explanation:

Corner points	Value of Z = 2x + 5y
P(0, 5)	Z = 2(0) + 5(5) = 25
Q(1, 5)	Z = 2(1) + 5(5) = 27
R(4, 2)	Z = 2(4) + 5(2) = 18 $\to$ Minimum
S(12, 0)	Z = 2(12) + 5(0) = 24

Thus, minimum value of Z occurs ar R(4, 2)

8.

(b) $\frac{2\pi}{9}$

Explanation:

$\cos^{-1}(\cos(680°))$

$= \cos^{-1}[\cos(720° - 40°)]$

$= \cos^{-1}[\cos(-40°)]$

$= \cos^{-1}[\cos(40°)]$

$= 40°$

$= \frac{2\pi}{9}$.

9.

(c) 0

Explanation:

If f is an odd function,
$\int_{-a}^{a} f(x) dx=0$
as, $\int_0^a f(x) dx=-\int_{-a}^0 f(x) dx$
f(x) = $\sin^5$ x
f(-x) = $\sin^5$ (-x)
Therefore, f(x) is odd number
$\int_{-\pi}^{\pi} \sin^5 x d x=0$

10.

(b) x = 2, y = 3
Explanation:
We have, $\begin{bmatrix}2x+y & 4x \\ 5x-7 & 4x\end{bmatrix}=\begin{bmatrix}7 & 7y-13 \\ y & x+6\end{bmatrix}$
⇒ 4x = x + 6 ⇒ x = 2
and 4x = 7y - 13
⇒ 8 = 7y - 13
⇒ y = 3

11. **(a)** the problem is to be re-evaluated
Explanation:
The optimisation of the objective function of a LPP is governed by the constraints. Therefore, if the constraints in a linear programming problem are changed, then the problem needs to be re-evaluated.

12.

(c) -1
Explanation:
We know that,
$|\vec{a}+\vec{b}+\vec{c}|^2=|\overrightarrow{a}^2|+|\vec{b}|^2+|\vec{c}|^2+2 \vec{a} \cdot \vec{b}+2 \vec{b} \cdot \vec{c}+2 \vec{c} \cdot \vec{a}$ ... (i)
Since,
$\vec{a}$ is perpendicular to $\vec{b}$
$\Rightarrow \vec{a} \cdot \vec{b}=0$
And according to question
$|\vec{a}|=|\vec{b}|=|\vec{c}|=1$
We can rewrite equation (i) as
$|\vec{a}+\vec{b}+\vec{c}|^2=|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+0+2 \cos \beta+2 \cos \alpha$
1 = 1 + 1 + 1 + 0 + 2 (cos α + cos β)
1=3+2$(cos\alpha + cos\beta)$
-2=2$(cos\alpha + cos\beta)$
⇒cos α + cosβ = -1

13. **(a)** 11
Explanation:
11

14.

(b) $\frac{1}{35}$
Explanation:
Consonants can be placed in 4! ways.
Vowels can be placed in between consonants in 4! ways
∴ Total no. of ways in which the consonants and vowels occur alternatively = $2 \times 4! \times 4!$
Also, total no. of ways in which the letters of the word 'ORIENTAL' can be arranged = 8!
∴ Required probability = $\frac{2 \times 4! \times 4!}{8!}=\frac{1}{35}$

15. **(a)** $y^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$

Explanation:

$y^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$

16.

(b) $\pm\frac{\hat{i}+\hat{j}}{\sqrt{2}}$

Explanation:

Since, unit vector perpendicular to both $\vec{a}$ and $\vec{b} = \pm\frac{a\times b}{|a\times b|}$

$\therefore \vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix}$

$= \hat{i}[1+1] - \hat{j}[-1-1] + q\hat{k}[1-1]$

$= 2\hat{i} + 2\hat{j} + 0\hat{k} = 2(\hat{i}+\hat{j})$

and $|\vec{a}\times\vec{b}| = \sqrt{4+4} = 2\sqrt{2}$

$\therefore$ Required unit vector

$= \pm\frac{2(\hat{i}+\hat{j})}{2\sqrt{2}} = \pm\frac{\hat{i}+\hat{j}}{\sqrt{2}}$

17.

(b) continuous $\forall$ x $\in$ R and not differentiable at x = 0

Explanation:

f(x) = x - |x| = g(x) + h(x)

where y(x) = x, h(x) = -|x|

As g(x) and h(x) are both continuous $\forall$ x $\in$ R

$\therefore$ f(x) is continuous $\forall$ x $\in$ R

And g(x) is differentiable $\forall$ x $\in$ R

but h(x) is not differentiable at x = 0

$\therefore$ f(x) = g(x) + h(x) is not differentiable at x = 0

18.

(b) $\frac{4}{3}$

Explanation:

$\frac{4}{3}$

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Let f(x) = x^2 - 8x + 17

$\therefore$ f'(x) = 2x - 8

So, f'(x) = 0, gives x = 4

Here x = 4 is the critical number

Now, f''(x) = 2 > 0 , $\forall$ x

So, x = 4 is the point of local minima.

$\therefore$ Minimum value of f(x) at x = 4,

f(4) = 4 $\times$ 4 - 8 $\times$ 4 + 17 = 1

Hence, we can say that both Assertion and Reason are true and Reason is the correct explanation of the Assertion.

20.

(c) A is true but R is false.

Explanation:

Assertion: Here, f : R → R is given by

f(x) = |x| = $\begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$

It is seen that f(-1) = |-1| = 1, f(1) = |1| = 1

Therefore, f(-1) = f(1) but -1 $\neq$ 1

Therefore, f is not one-one.

Now, consider -1 $\in$ R

It is known that f(x) = |x| is always non-negative

Thus, there does not exist any element x in domain R such that f (x) = |x| = -1.

Therefore, f is not onto.

Hence, the modulus function is neither one-one nor onto.

Reason: f : R → R, f(x) = $\begin{cases} 1, & \text{if} \quad x > 0 \\ 0, & \text{if} \quad x = 0 \\ -1, & \text{if} \quad x < 0 \end{cases}$

It is seen that f(1) = f(2) = 1 but 1 $\neq$ 2.

Therefore, f is not one-one

Now, as f(x) takes only three values (1, 0 or -1), therefore for the element -2 in codomain R, there does not exist any x in domain R such that f(x) = -2

Therefore, f is not onto. Hence, the Signum function is neither one-one nor onto.

Section B

21. Let $\sin^{-1}\left(\frac{-1}{2}\right) = y$

$\Rightarrow \sin y = -\frac{1}{2}$

$\Rightarrow \sin y = -\sin\frac{\pi}{6}$

$\Rightarrow \sin y = \sin\left(-\frac{\pi}{6}\right)$

Since, the principal value branch of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Therefore, principal value of $\sin^{-1}\left(\frac{-1}{2}\right)$ is $-\frac{\pi}{6}$.

OR

Let $\cos^{-1}\left(\frac{1}{2}\right) = x$. Then, $\cos x = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right)$.

$\therefore \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

Let $\sin^{-1}\left(\frac{1}{2}\right) = y$. Then, $\sin y = \frac{1}{2} = \sin\left(\frac{\pi}{6}\right)$.

$\therefore \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

$\therefore \cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} + \frac{2\pi}{6} = \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$

22. The function is f (x) = sin x

Then, $f'(x) = cosx$

Since for each x ϵ $\left(\frac{\pi}{2}, \pi\right)$, cos x < 0, we have $f'(x) < 0$

Therefore, f is strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$.

23. Here,

$y = \frac{2}{3}x^3 + 1$

Differentiating both sides with respect to t,

$\Rightarrow \frac{dy}{dt} = 2x^2\frac{dx}{dt}$

$\Rightarrow 2\frac{dx}{dt} = 2x^2\frac{dx}{dt}\left[\because \frac{dy}{dt} = 2\frac{dx}{dt}\right]$

$\Rightarrow x = \pm 1$

Substituting the value of x = 1 and x= -1 in y = $\frac{2}{3}x^3$ + 1, we get

$\Rightarrow y = \frac{5}{3}$ and $y = \frac{1}{3}$

So , the points are $\left(1, \frac{5}{3}\right)$ and $\left(-1, \frac{1}{3}\right)$

OR

Given $R(x) = 13x^2 + 26x + 15$

Now, $MR = \frac{d}{dx}(R(x)) = 26x + 26$

$MR]_{x=17} = 25 \times 17 + 26$

= 425 + 26

= 451

24. We have

$I = \int e^x \cdot \left\{\frac{2+\sin 2x}{1+\cos 2x}\right\} dx$

$= \int e^x \cdot \left\{\frac{2}{(1+\cos 2x)} + \frac{\sin 2x}{(1+\cos 2x)}\right\} dx$

$= \int e^x \cdot \left\{\frac{2}{2\cos^2 x} + \frac{2\sin x \cos x}{2\cos^2 x}\right\} dx$ = $\int e^x.|\sec^2 x + \tan x|$ dx

= $\int e^x.(\tan x + \sec^2 x)$ dx

= $\int e^x .\{f(x) + f'(x)\}$ dx where f(x) = tan x a f'(x) = $\sec^2 x$

= e^x f(x) + C = e^x tan x + C

25. Given: $A = \begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{bmatrix}$

$\Rightarrow |A| = \begin{vmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{vmatrix}$

Expanding along first row, $1\begin{vmatrix} 1 & -3 \\ 4 & -9 \end{vmatrix} - 1\begin{vmatrix} 2 & -3 \\ 5 & -9 \end{vmatrix} + (-2)\begin{vmatrix} 2 & 1 \\ 5 & 4 \end{vmatrix}$

= {-9(-12)} - {-18 - (-15)} - 2(8 - 5)

= -9 + 12 - (-18 + 15) -2(3)

= 3(-3) - 6

= 3 + 3 - 6 = 0

Section C

26. let the given integral be

l = $\int \frac{3\sin x + 2\cos x}{3\cos x + 2\sin x} dx$

Let 3 sin x + 2 cos x = $\lambda \frac{d}{dx}$ (3 cos x + 2 sin x) + μ (3 cos x + 2 sin x)

i.e. 3 sin x + 2 cos x = λ (- 3 sin x + 2 cos x) + μ(3 cos x + 2 sin x)

Comparing the coefficients of sin x and cos x on both sides, we get

- 3λ + 2μ = 3 and 2λ + 3μ = 2 $\Rightarrow \mu = \frac{12}{13}$ and $\lambda = -\frac{5}{13}$

$\therefore \quad I = \int \frac{\lambda(-3\sin x + 2\cos x) + \mu(3\cos x + 2\sin x)}{3\cos x + 2\sin x} dx$

$\Rightarrow \quad I = \mu \int 1 \cdot dx + \lambda \int \frac{-3\sin x + 2\cos x}{3\cos x + 2\sin x} dx$

$\Rightarrow \quad I = \mu x + \lambda \int \frac{dt}{t}$, where t = 3 cos x + 2 sin x

$\Rightarrow \quad I = \mu$ x + λ log |t| + C

= $\frac{12}{13}x - \frac{5}{13}$ log |3 cos x + 2 sin x| + C

27. A white ball and a red ball can be drawn in three mutually exclusive ways:

i. Selecting bag I and then drawing a white and a red ball from it

ii. Selecting bag II and then drawing a white and a red ball from it

iii. Selecting bag III and then drawing a white and a red ball from it

Consider the following events:

E_1 = Selecting bag I

E_2 = Selecting bag II

E_3 = Selecting bag II

A = Drawing a white and a red ball

It is given that one of the bags is selected randomly.

$\therefore P(E_1) = \frac{1}{3}$

P(E_2) = $\frac{1}{3}$

P(E_3) = $\frac{1}{3}$

Now,

$P\left(\frac{A}{E_1}\right) = \frac{{}^1C_1 \times {}^3C_1}{{}^6C_2} = \frac{3}{15}$
$P\left(\frac{A}{E_2}\right) = \frac{{}^2C_1 \times {}^1C_1}{{}^4C_2} = \frac{2}{6}$
$P\left(\frac{A}{E_3}\right) = \frac{{}^4C_1 \times {}^3C_1}{{}^{12}C_2} = \frac{12}{66}$

Using the law of total probability, we get

Required probability = P(A) = $P(E_1)P(\frac{A}{E_1}) + P(E_2)P(\frac{A}{E_2}) + P(E_3)P(\frac{A}{E_3})$

$= \frac{1}{3} \times \frac{3}{15} + \frac{1}{3} \times \frac{2}{6} + \frac{1}{3} \times \frac{12}{66}$
$= \frac{1}{15} + \frac{1}{9} + \frac{2}{33}$
$= \frac{33+55+30}{495} = \frac{118}{495}$

28. Given $I = \int \frac{2x}{(1+x^2)(x^2+3)} dx$

Put $x^2 = t \Rightarrow 2xdx = dt$

$\therefore I = \int \frac{dt}{(t+1)(3+t)}$

Using partial fractions,

$\frac{1}{(t+1)(3+t)} = \frac{A}{(1+t)} + \frac{B}{(3+t)}$ ...(i)

$\Rightarrow 1 = A(3 + t) + B(1 + t)$

Putting t = -3, we get

$\Rightarrow 1 = -2B \Rightarrow B = -\frac{1}{2}$

Putting t = -1, we get

$\Rightarrow 1 = 2A \Rightarrow A = \frac{1}{2}$

On putting A = $\frac{1}{2}$ and B = $-\frac{1}{2}$ in Eq. (i), we get

$\frac{1}{(1+t)(3+t)} = \frac{\frac{1}{2}}{1+t} + \frac{\frac{(-1)}{2}}{3+t}$

Integrating both sides w.r.t. to t ,

$\Rightarrow \int \frac{1}{(1+t)(3+t)} dt = \frac{1}{2} \int \frac{1}{1+t} dt - \frac{1}{2} \int \frac{1}{3+t} dt$

$= \frac{1}{2} \log|1+t| - \frac{1}{2} \log|3+t| + C$

$= \frac{1}{2} \log|1+x^2| - \frac{1}{2} \log|3+x^2| + C[\text{ put } t = x^2]$

$\therefore \quad I = \frac{1}{2} \log\left|\frac{1+x^2}{3+x^2}\right| + C$ [$\because$ log m - log n = log$\frac{m}{n}$]

OR

We have,

$\frac{1}{1+\cot x} = \frac{1}{1+\frac{\cos x}{\sin x}} = \frac{\sin x}{\sin x+\cos x}$

$\therefore \int_0^{\frac{\pi}{2}} \frac{1}{1+\cot x} dx = \int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x+\cos x} dx$

Let

$I = \int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x+\cos x} dx$ ..(i)

so, using property of defnite integrals we have

$I = \int_0^{\frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)} dx \; \left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]\right]$

$= \int_0^{\frac{\pi}{2}} \frac{\cos x}{\sin x+\cos x} dx$ ..(ii)

Adding (i) & (ii), we get

$2I = \int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x+\cos x} dx + \int_0^{\frac{\pi}{2}} \frac{\cos x}{\sin x+\cos x} dx$

$= \int_0^{\frac{\pi}{2}} \frac{\sin x+\cos x}{\sin x+\cos x} dx$

$= \int_0^{\frac{\pi}{2}} dx$

$= [x]_0^{\frac{\pi}{2}}$

$= \left[\frac{\pi}{2} - 0\right]$

$\Rightarrow \; I = \frac{\pi}{4}$

29. $\left(x^2+y^2\right) dy = xydx$

$\Rightarrow \int \frac{x}{y} dy + \int \frac{y}{x} dy = \int dx$

$\Rightarrow x \log y + \frac{y^2}{2x} = x + c$

Now, at x = 1; y = e

$x \log y + \frac{y^2}{2x} = x + c \Rightarrow x + \frac{e^2}{2} = x + c \Rightarrow c = \frac{e^2}{2}$

Now at x = x_0; y = e

$x_0 \log e + \frac{e^2}{2x_0} = x_0 + \frac{e^2}{2} \Rightarrow \frac{e^2}{2x_0} = \frac{e^2}{2} \Rightarrow x_0 = 1$

OR

The given differential equation may be written as

$\frac{dy}{dx} = \frac{3xy}{(y^2-x^2)}$ ...(i)

On dividing the Nr and Dr of RHS of (i) by x^2, we have,

$\frac{dy}{dx} = \frac{3\left(\frac{y}{x}\right)}{\left\{\left(\frac{y}{x}\right)^2 - 1\right\}} = f\left(\frac{y}{x}\right)$

Therefore, the given differential equation is homogeneous.

Putting y = vx and $\frac{dy}{dx} = v + x\frac{dv}{dx}$ in (i), we have,

$v + x\frac{dv}{dx} = \frac{3v}{(v^2-1)}$

$\Rightarrow \quad x\frac{dv}{dx} = \left\{\frac{3v}{(v^2-1)} - v\right\} = \frac{(4v-v^3)}{(v^2-1)}$

$\Rightarrow \quad \frac{(v^2-1)}{(4v-v^3)} dv = \frac{1}{x} dx$

$\Rightarrow \int \frac{(v^2-1)}{v(2-v)(2+v)} dv = \int \frac{1}{x} dx$ ...(ii)

Let $\frac{(v^2-1)}{v(2-v)(2+v)} = \frac{A}{v} + \frac{B}{(2-v)} + \frac{C}{(2+v)}$

Therefore, $(v^2 - 1) \equiv$ A(2 - v)(2 + v) + Bv (2 + v) + Cv(2 - v) .. (ii)

Putting v = 0 on each side of (iii), we get A = $\frac{-1}{4}$

Putting u = 2 on each side of (iii), we get B = $\frac{3}{8}$

Putting v = -2 on each side of (iii), we get C = $\frac{-3}{8}$

$\therefore \frac{(v^2-1)}{v(2-v)(2+v)} = \frac{-1}{4v} + \frac{3}{8(2-v)} - \frac{3}{8(2+v)}$ ...(iv)

Putting these values from (iv) in (ii), we have,

$-\frac{1}{4}\int \frac{dv}{v} + \frac{3}{8}\int \frac{dv}{(2-v)} - \frac{3}{8}\int \frac{dv}{(2+v)} = \int \frac{1}{x} dx$

$\Rightarrow \int \frac{1}{x} dx + \frac{1}{4}\int \frac{dv}{v} + \frac{3}{8}\int \frac{-dv}{(2-v)} + \frac{3}{8}\int \frac{dv}{(2+v)}$ = log $|C_1|$ where C_1 is an arbitrary constant

$\Rightarrow$ log |x| + $\frac{1}{4}$ log |v| + $\frac{3}{8}$ log |2 - v| + $\frac{3}{8}$ log |2 + v| = log $|C_1|$

$\Rightarrow$ 8 log |x| + 2 log |v| + 3 log |2 - v| + 3 log |2 + v| = 8 log $|C_1|$

$\Rightarrow$ log $|x^8v^2(2 - v)^3(2 + v)^3|$ = log $(C_1^8\}$

$\Rightarrow$ $|x^8v^2(2 - v)^3(2 + v)^3| = C_1^8$ = C (say)

$\Rightarrow \quad x^6y^2\left(2-\frac{y}{x}\right)^3\left(2+\frac{y}{x}\right)^3$ = C

This is the required solution of given differential equation.

30. Given,

$\vec{a}+\vec{b}+\vec{c} = \overrightarrow{0} \Rightarrow \vec{a}+\vec{b} = -\vec{c}$

$\Rightarrow (\vec{a}+\vec{b}) \times \vec{b} = (-\vec{c}) \times \vec{b}$

$\Rightarrow (\vec{a}\times\vec{b}) + (\vec{b}\times\vec{b}) = (-\vec{c}) \times \vec{b}$ [by the distributive law]

$\Rightarrow (\vec{a}\times\vec{b}) + \overrightarrow{0} = (\vec{b}\times\vec{c})$ [$\because \quad \vec{b}\times\vec{b} = \overrightarrow{0}$ and $(-\vec{c})\times\vec{b} = \vec{b}\times\vec{c}$]

$\Rightarrow \vec{a}\times\vec{b} = \vec{b}\times\vec{c}$ ...(i)

Also, $\vec{a}+\vec{b}+\vec{c} = \overrightarrow{0} \Rightarrow \vec{b}+\vec{c} = -\vec{a}$

$\Rightarrow (\vec{b}+\vec{c}) \times \vec{c} = (-\vec{a}) \times \vec{c}$

$\Rightarrow (\vec{b}\times\vec{c}) + (\vec{c}\times\vec{c}) = (-\vec{a}) \times \vec{c}$ [by the distributive law]

$\Rightarrow (\vec{b} \times \vec{c}) + \vec{0} = \vec{c} \times \vec{a}$ $[\because \vec{c} \times \vec{c} = \vec{0}$ and $(-\vec{a}) \times \vec{c} = \vec{c} \times \vec{a}]$

$\Rightarrow \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$ ...(ii)

From (i) and (ii), we get

$\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$

OR

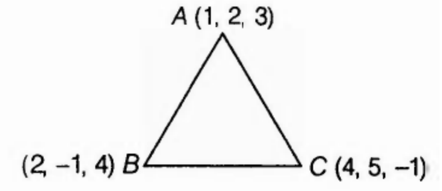
Given , ΔABC, whose vertices are A (1, 2, 3), B(2, -1, 4) and C(4, 5, -1).

Let the position vectors of the vertices A, B and C of ΔABC is

$\overrightarrow{OA} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\overrightarrow{OB} = 2\hat{i} - \hat{j} + 4\hat{k}$ and

$\overrightarrow{OC} = 4\hat{i} + 5\hat{j} - \hat{k}$



We know that , $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= (2\hat{i} - \hat{j} + 4\hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k})$

$= \hat{i} - 3\hat{j} + \hat{k}$

also , $\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA}$

$= (4\hat{i} + 5\hat{j} - \hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k})$

$= (3\hat{i} + 3\hat{j} - 4\hat{k})$

Now, $\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 1 \\ 3 & 3 & -4 \end{vmatrix}$

$= \hat{i}(12 - 3) - \hat{j}(-4 - 3) + \hat{k}(3 + 9)$

$= 9\hat{i} + 7\hat{j} + 12\hat{k}$

$|\overrightarrow{AB} \times \overrightarrow{AC}| = \sqrt{(9)^2 + (7)^2 + (12)^2}$

$= \sqrt{81 + 49 + 144}$

$= \sqrt{274}$

$\therefore$ Area of $\Delta ABC = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$

$= \frac{1}{2}\sqrt{274}$

$= \frac{\sqrt{274}}{2}$

$\therefore$ $Area = \frac{\sqrt{274}}{2}$ sq units.

31. According to the question,we have to prove that $\frac{dy}{dx} = -\frac{1}{(1+x)^2}$ if $x\sqrt{1+y} + y\sqrt{1+x} = 0$

where x≠y.

we shall first write y in terms of x explicitly i.e y=f(x)

Clearly, $x\sqrt{1+y} = -y\sqrt{1+x}$

Squaring both sides,we get,

$x^2(1 + y) = y^2 (1 + x)$

$\Rightarrow x^2 + x^2y = y^2(1 + x)$

$\Rightarrow x^2 - y^2 = y^2x - x^2y$

$\Rightarrow$(x - y)(x + y) = -xy(x - y)

$\Rightarrow$(x - y)(x + y) + xy(x - y) = 0

$\Rightarrow$(x - y)(x + y + xy) = 0

$\therefore$Either, x - y = 0 or x + y + xy = 0

Now, x - y = 0 $\Rightarrow$ x = y

But, it is given that x $\neq$ y

So, it is a contradiction

Therefore, x - y = 0 is rejected.

Now, consider y + xy + x = 0

$\Rightarrow$ y(1 + x) = -x $\Rightarrow$y = $\frac{-x}{1+x}$ ....(i)

Therefore,on differentiating both sides w.r.t x, we get,

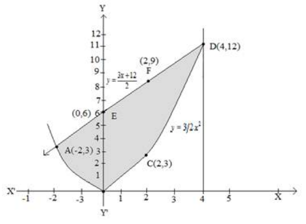
$\frac{dy}{dx} = \frac{(1+x)\times\frac{d}{dx}(-x)-(-x)\times\frac{d}{dx}(1+x)}{(1+x)^2}$[By using quotient rule of derivative]

$\Rightarrow \frac{dy}{dx} = \frac{(1+x)(-1)+x(1)}{(1+x)^2}$

$\Rightarrow \frac{dy}{dx} = \frac{-1-x+x}{(1+x)^2}$

$\therefore \frac{dy}{dx} = \frac{-1}{(1+x)^2}$

Section D

32. 

$4y = 3x^2$ ......(1)

2y = 3x + 12......(2)

From (2), $y = \frac{3x+12}{2}$

Using this value of y in (1), we get,

x^2 - 6x - 8 = 0

$\Rightarrow$ (x + 2) (x - 4) = 0

$\Rightarrow$ x = -2, 4

From (2),

When, x = -2, y = 3

When, x = 4, y = 12

Thus, points of intersection are, (-2, 3) and (4, 12).

Area $= \int_{-2}^{4} \frac{3x+12}{2}dx - \int_{-2}^{4} \frac{3}{4}x^2dx$

$= \frac{1}{2}[\frac{3x^2}{2} + 12x]_{-2}^{4} - \frac{3}{4}[\frac{x^3}{3}]_{-2}^{4}$

$\frac{1}{2}[(24+48)-(6-24)] - \frac{1}{4}[64-(-8)]$

= 45 - 18 = 27 sq units.

33. Here R is a relation on $N \times N$, defined by (a, b) R (c, d) $\Leftrightarrow$ a + d = b + c for all (a, b), (c, d) $\in N \times N$

We shall show that R satisfies the following properties

i. Reflexivity:

We know that a + b = b + a for all a, b $\in$ N.

$\therefore$ (a, b) R (a, b) for all (a, b) $\in (N \times N)$

So, R is reflexive.

ii. Symmetry:

Let (a, b) R (c, d). Then,

(a, b) R (c, d) $\Rightarrow$ a + d = b + c

$\Rightarrow$ c + b = d + a

$\Rightarrow$ (c, d) R (a, b).

$\therefore$ (a, b) R (c, d) $\Rightarrow$ (c, d) R (a, b) for all (a, b), (c, d) $\in N \times N$

This shows that R is symmetric.

iii. Transitivity:

Let (a, b) R (c, d) and (c, d) R (e, f). Then,

(a, b) R (c, d) and (c, d) R (e, f)

$\Rightarrow$ a + d = b + c and c + f = d + e

$\Rightarrow$ a + d + c + f = b + c + d + e

$\Rightarrow$ a + f = b + e

$\Rightarrow$ (a, b) R (e, f).

Thus, (a, b) R (c, d) and (c, d) R (e, f) ⇒ (a, b) R (e, f)
This shows that R is transitive.
∴ R is reflexive, symmetric and transitive
Hence, R is an equivalence relation on $N \times N$

OR

f is one-one: For any x, y ∈ R - {+1}, we have $f(x) = f(y)$
$\Rightarrow \frac{x}{1+|x|} = \frac{y}{|y|+1}$
⇒ xy + x = xy + y
⇒ x = y
Therefore, f is one-one function.
If f is one-one, let y = R - {1}, then f(x) = y
$\Rightarrow \frac{x}{x+1} = y$
$\Rightarrow x = \frac{y}{1-y}$
It is cleat that x ∈ R for all y = R - {1}, also x = ≠ -1
Because x = -1
$\Rightarrow \frac{y}{1-y} = -1$
⇒ y = -1 + y
which is not possible.
Thus for each R - {1} there exists $x = \frac{y}{1-y}$ ∈ R - {1} such that
$$f(x) = \frac{x}{x+1} = \frac{\frac{y}{1-y}}{\frac{y}{1-y}+1} = y$$
Therefore f is onto function.

34. We have
$$A = \begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 1 & -6 & -4 \\ 3 & 8 & 6 \\ 5 & 3 & 5 \end{bmatrix}$$
∴(A + A ') = $\begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix} + \begin{bmatrix} 1 & -6 & -4 \\ 3 & 8 & 6 \\ 5 & 3 & 5 \end{bmatrix}$
$$= \begin{bmatrix} 1+1 & 3+(-6) & 5+(-4) \\ -6+3 & 8+8 & 3+6 \\ -4+5 & 6+3 & 5+5 \end{bmatrix} = \begin{bmatrix} 2 & -3 & 1 \\ -3 & 16 & 9 \\ 1 & 9 & 10 \end{bmatrix}$$
Suppose P = $\frac{1}{2}(A+A') = \frac{1}{2}\begin{bmatrix} 2 & -3 & 1 \\ -3 & 16 & 9 \\ 1 & 9 & 10 \end{bmatrix}$ = $\begin{bmatrix} 1 & \frac{-3}{2} & \frac{1}{2} \\ \frac{-3}{2} & 8 & \frac{9}{2} \\ \frac{1}{2} & \frac{9}{2} & 5 \end{bmatrix}$
And, we have (A - A') = $\begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix} - \begin{bmatrix} 1 & -6 & -4 \\ 3 & 8 & 6 \\ 5 & 3 & 5 \end{bmatrix}$
$$= \begin{bmatrix} 1-1 & 3-(-6) & 5-(-4) \\ -6-3 & 8-8 & 3-6 \\ -4-5 & 6-3 & 5-5 \end{bmatrix} = \begin{bmatrix} 0 & 3+6 & 5+4 \\ -9 & 0 & -3 \\ -9 & 3 & 0 \end{bmatrix}$$
$$= \begin{bmatrix} 0 & 9 & 9 \\ -9 & 0 & -3 \\ -9 & 3 & 0 \end{bmatrix}$$
Let Q =$\frac{1}{2}(A-A') = \frac{1}{2}\begin{bmatrix} 0 & 9 & 9 \\ -9 & 0 & -3 \\ -9 & 3 & 0 \end{bmatrix}$ = $\begin{bmatrix} 0 & \frac{9}{2} & \frac{9}{2} \\ \frac{-9}{2} & 0 & \frac{-3}{2} \\ \frac{-9}{2} & \frac{3}{2} & 0 \end{bmatrix}$
$$P' = \begin{bmatrix} 1 & \frac{-3}{2} & \frac{1}{2} \\ \frac{-3}{2} & 8 & \frac{9}{2} \\ \frac{1}{2} & \frac{9}{2} & 5 \end{bmatrix} = \begin{bmatrix} 1 & \frac{-3}{2} & \frac{1}{2} \\ \frac{-3}{2} & 8 & \frac{9}{2} \\ \frac{1}{2} & \frac{9}{2} & 5 \end{bmatrix} = P$$
∴ P is symmetric.

And $Q' = \begin{bmatrix} 0 & \frac{9}{2} & \frac{9}{2} \\ \frac{-9}{2} & 0 & \frac{-3}{2} \\ \frac{-9}{2} & \frac{3}{2} & 0 \end{bmatrix} = \begin{bmatrix} 0 & \frac{-9}{2} & \frac{-9}{2} \\ \frac{9}{2} & 0 & \frac{3}{2} \\ \frac{9}{2} & \frac{-3}{2} & 0 \end{bmatrix} = -Q$

$\therefore$ Q is skew-symmetric.

Now, (P + Q) = $= \begin{bmatrix} 1 & \frac{-3}{2} & \frac{1}{2} \\ \frac{-3}{2} & 8 & \frac{9}{2} \\ \frac{1}{2} & \frac{9}{2} & 5 \end{bmatrix} + \begin{bmatrix} 0 & \frac{9}{2} & \frac{9}{2} \\ \frac{-9}{2} & 0 & \frac{-3}{2} \\ \frac{-9}{2} & \frac{3}{2} & 0 \end{bmatrix}$

$= \begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix} = A$

Therefore, A = P + Q

where P is symmetric and Q is skew-symmetric.

35. Let AC = x, BC = y and r be the radius of circle.

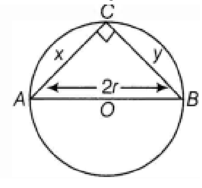
Also, $\angle C = 90^\circ$ [$\because$ angle made in semi-circle is 90°]

In ΔABC, we have

$(AB)^2 = (AC)^2 + (BC)^2$

$(2r)^2 = (x)^2 + (y)^2$

$4r^2 = x^2 + y^2$ ...(I)



We know that,

Area of ΔABC, $(A) = \frac{1}{2}\, x \cdot y$

On squaring both sides, we get

$A^2 = \frac{1}{4}x^2y^2$

Let $A^2 = S$

Then, $S = \frac{1}{4}x^2y^2$

$\Rightarrow \quad S = \frac{1}{4}x^2\left(4r^2 - x^2\right)$ [from Eq. (i)]

$\Rightarrow \quad S = \frac{1}{4}\left(4x^2r^2 - x^4\right)$

On differentiating both sides w.r.t.x, we get

$\frac{dS}{dx} = \frac{1}{4}\left(8r^2x - 4x^3\right)$

For maxima or minima, put $\frac{dS}{dx} = 0$

$\therefore \quad \frac{1}{4}\left(8r^2x - 4x^3\right) = 0$

$8r^2x = 4x^3$

$8r^2 = 4x^2$

$x^2 = 2r^2$

$x = \sqrt{2}r$

From Eq. (i), we get,

$y^2 = 4r^2 - 2r^2 = 2r^2 \Rightarrow y = \sqrt{2}r$

Here, x = y so triangle is an isosceles.

Also, $\frac{d^2S}{dx^2} = \frac{d}{dx}\left[\frac{1}{4}\left(8r^2x - 4x^3\right)\right] = \frac{1}{4}\left(8r^2 - 12x^2\right)$

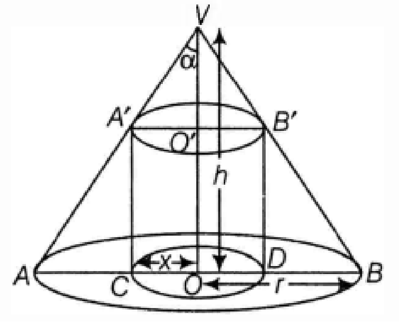
$= 2r^2 - 3x^2$ m

At $x = \sqrt{2}r, \frac{d^2S}{dx^2} = 2r^2 - 3(2r^2) = -4r^2 < 0$

Therefore, Area of the triangle is maximum when it is an isosceles triangle.

OR

Let VAB be the cone of base radius r, height h and radius of base of the inscribed cylinder be x.



Now, we observe that

$\Delta VOB \sim \Delta B'DB \Rightarrow \frac{VO}{B'D} = \frac{OB}{DB}$ [∵ if the triangles are similar, then their sides are proportional]

$\Rightarrow \quad \frac{h}{B'D} = \frac{r}{r-x}$

$\Rightarrow \quad B'D = \frac{h(r-x)}{r}$

Let C be the curved surface area of cylinder. Then,

C = 2π(OC)(B'D)

$\Rightarrow \quad C = \frac{2\pi x h(r-x)}{r} = \frac{2\pi h}{r}\left(rx - x^2\right)$

Therefore,on differentiating both sides w.r.t. x, we get,

$\frac{dC}{dx} = \frac{2\pi h}{r}(r - 2x)$

For Maxima or Minima, put $\frac{dC}{dx} = 0$

$\Rightarrow \quad \frac{2\pi h}{r}(r - 2x) = 0$

$\Rightarrow \quad r - 2x = 0 \Rightarrow r = 2x$

$\therefore x = \frac{r}{2}$

Therefore, radius of cylinder is half of that of cone.

Also, $\frac{d^2C}{dx^2} = \frac{d}{dx}\left[\frac{2\pi h(r-2x)}{r}\right]$

$= \frac{2\pi h}{r}(-2) = \frac{-4\pi h}{r} < 0$ as $h, r > 0$

Therefore, C is maximum or greatest.

Hence, C is greatest at x = $\frac{r}{2}$

Section E

36. i. Let E_1: Ajay (A) is selected, E_2: Ramesh (B) is selected, E_3: Ravi (C) is selected

Let A be the event of making a change

$P(E_1) = \frac{4}{7}, \ P(E_2) = \frac{1}{7}, \ P(E_3) = \frac{2}{7}$

$P(A/E_1) = 0.3$, $P(A/E_2) = 0.8$, $P(A/E_3) = 0.5$

$P(E_1/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\cdot P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)}$

$= \frac{\frac{4}{7}\times 0.3}{\frac{4}{7}\times 0.3+\frac{1}{7}\times 0.8+\frac{2}{7}\times 0.5} = \frac{\frac{1.2}{7}}{\frac{1.2}{7}+\frac{0.8}{7}+\frac{1}{7}} = \frac{\frac{1.2}{7}}{\frac{3}{7}}$

$= \frac{1.2}{3} = \frac{12}{30} = \frac{2}{5}$

ii. Let E_1: Ajay(A) is selected, E_2: Ramesh(B) is selected, E_3: Ravi (C) is selected

Let A be the event of making a change

$P(E_1) = \frac{4}{7}, \ P(E_2) = \frac{1}{7}, \ P(E_3) = \frac{2}{7}$

$P(A/E_1) = 0.3$, $P(A/E_2) = 0.8$, $P(A/E_3) = 0.5$

$P(E_2/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\cdot P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)}$

$= \frac{\frac{1}{7}\times 0.8}{\frac{4}{7}\times 0.3+\frac{1}{7}\times 0.8+\frac{2}{7}\times 0.5} = \frac{\frac{0.8}{7}}{\frac{1.2}{7}+\frac{0.8}{7}+\frac{1}{7}} = \frac{\frac{0.8}{7}}{\frac{3}{7}}$

$= \frac{0.8}{3} = \frac{8}{30} = \frac{4}{15}$

iii. Let E_1: Ajay (A) is selected, E_2: Ramesh (B) is selected, E_3: Ravi (C) is selected

Let A be the event of making a change

$P(E_1) = \frac{4}{7}, \ P(E_2) = \frac{1}{7}, \ P(E_3) = \frac{2}{7}$

$P(A/E_1) = 0.3$, $P(A/E_2) = 0.8$, $P(A/E_3) = 0.5$

$P(E_3/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\cdot P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)}$

$= \frac{\frac{2}{7}\times 0.5}{\frac{4}{7}\times 0.3+\frac{1}{7}\times 0.8+\frac{2}{7}\times 0.5} = \frac{\frac{1}{7}}{\frac{1.2}{7}+\frac{0.8}{7}+\frac{1}{7}} = \frac{1}{3}$

OR

Let E_1: Ajay (A) is selected, E_2: Ramesh (B) is selected, E_3: Ravi (C) is selected

Let A be the event of making a change

$P(E_1) = \frac{4}{7},\ P(E_2) = \frac{1}{7},\ P(E_3) = \frac{2}{7}$

$P(A/E_1) = 0.3$, $P(A/E_2) = 0.8$, $P(A/E_3) = 0.5$

Ramesh or Ravi

$\Rightarrow P(E_2/A) + P(E_3/A) = \frac{4}{15} + \frac{1}{3} = \frac{9}{15} = \frac{3}{5}$

37. i. The line along which motorcycle A is running is,

$\vec{r} = \lambda(\hat{i} + 2\hat{j} - \hat{k})$, Which can be rewritten as

$(x\hat{i} + y\hat{j} + z\hat{k}) = \lambda\hat{i} + 2\lambda\hat{j} - \lambda\hat{k}$

$\Rightarrow x = \lambda, y = 2\lambda, z = -\lambda$

$\Rightarrow \frac{x}{1} = \lambda, \frac{y}{2} = \lambda, \frac{z}{-1} = \lambda$

Thus, the required cartesian equations is $\frac{x}{1} = \frac{y}{2} = \frac{z}{-1}$

ii. Here, $\vec{a}_1 = 0\hat{i} + 0\hat{j} + 0\hat{k}, \vec{a}_2 = 3\hat{i} + 3\hat{j},\ \vec{b}_1 = \hat{i} + 2\hat{j} - \hat{k},$

$\vec{b}_2 = 2\hat{i} + \hat{j} + \hat{k}$

$\therefore \vec{a}_2 - \vec{a}_1 = 3\hat{i} + 3\hat{j}$

and $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 2 & 1 & 1 \end{vmatrix} = 3\hat{i} - 3\hat{j} - 3\hat{k}$

Now, $(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = (3\hat{i} + 3\hat{j}) \cdot (3\hat{i} - 3\hat{j} - 3\hat{k})$

$= 9 - 9 = 0$

Hence, shortest distance between the given lines is 0.

iii. Since, direction cosies of a line are $\frac{k}{3}, \frac{k}{3}$, and $\frac{k}{3}$,

$\therefore l = \frac{k}{3}, m = \frac{k}{3}$, and $n = \frac{k}{3}$,

$\frac{k^2}{9} + \frac{k^2}{9} + \frac{k^2}{9} = 1$

$\Rightarrow \frac{3k^2}{9} = 1$

$\Rightarrow k^2 = 3$

$\Rightarrow k = \pm\sqrt{3}$

OR

The line joining the given point is:

$\frac{x-1}{1} = \frac{y-3}{-2} = \frac{z-6}{2} = \lambda$

Let $(\lambda + 1, -2\lambda + 3, 2\lambda + 6)$ be a point on the line.

Given, the points meets at XY - plane, so Z-coordinate will be zero.

$\therefore 2\lambda + 6 = 0$

$\Rightarrow \lambda = -3$

$\therefore$ Points is (-2, 9, 0)

38. i. Let number of pairs of earing = x and number of Necklaces = y

As per the given information

$x, y \geq 0$

$0.5x + y \leq 10$

$x + y \leq 15$

Profit function = $Z = 30x + 40y$

ii. Let number of pairs of earing = x and number of Necklaces = y

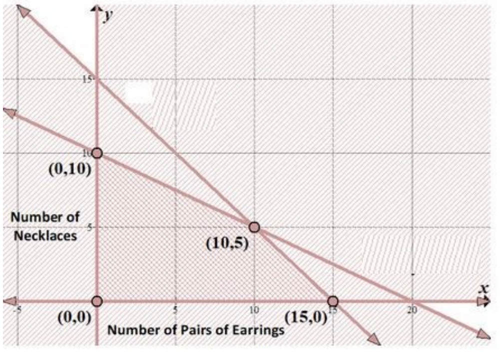
As per the given information

$x, y \geq 0$

$0.5x + y \leq 10$

$x + y \leq 15$

Profit function = Z = 30x + 40y



iii. From graph corner points are (0, 0), (0, 10), (10, 5) and (15, 0).

corner points	**maximum profit = Z = 30x + 40y**
(0, 0)	Z = 0
(0, 10)	Z = ₹400
(10, 5)	**Z = ₹500**
(15, 0)	Z = ₹450

Hence profit is maximum when x = number of pair of Earings = 10 and y = Number of Neckleses

OR

When x = 5 and y = 5

Z = 30x + 40y = 150 + 200 = ₹350